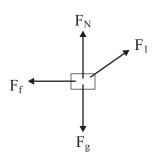
Mechanics

2007 C

1. (a)



(b)
$$\Sigma F_y = F_N + F_{1y} - F_g = ma_y = 0$$

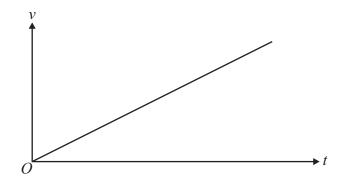
$$F_{N} = mg - F_{1} \sin \theta$$

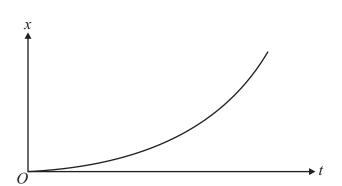
(c)
$$\Sigma F_x = F_{1x} - F_f = ma_1$$

 $F_1 \cos\theta - \mu F_N = ma_1$
 $\mu F_N = F \cos\theta_1 - ma_1$
 $\mu (mg - F_1 \sin\theta) = F_1 \cos\theta - ma_1$

$$\mu = \frac{\left(F_1 \cos \theta - ma_1\right)}{\left(mg - F_1 \sin \theta\right)}$$

(d)





(e)
$$\Sigma F_y = F_N + F_{1x} - F_g = ma_y = 0$$

 $0 + F_1 \sin \theta - mg = 0$
 $F_1 \sin \theta = mg$

$$F_1 = \frac{mg}{\sin\theta}$$

$$\sum F_x = F_{1x} - F_f = ma_{\text{max}}$$

$$\Sigma F_{x} = F_{1x} - F_{f} = ma_{\text{max}}$$

$$F_{1} \cos \theta - 0 = ma_{\text{max}}$$

$$a_{\text{max}} = \frac{F_{1} \cos \theta}{m} = F_{1} \frac{\cos \theta}{m} = \frac{mg}{\sin \theta} \frac{\cos \theta}{m}$$

$$a_{\text{max}} = g \cot \theta$$

Mechanics 2007 C

2. (a)
$$v = \frac{2\pi r}{T}$$

$$r_{orbit} = \frac{vT}{2\pi} = \frac{(3.40 \times 10^3 \text{ m/s})(7.08 \times 10^3 \text{ m/s})}{2\pi}$$

$$r_{orbit} = 3.83 \times 10^6 \text{ m}$$

(b)
$$\Sigma F = F_g = ma_c$$

$$G \frac{m_M m_S}{r_{M-S}^2} = m_S \frac{v^2}{r_{orbit}}$$
Note: r_{M-S} is the distance from the center of mars to the satellite, while $r_{M-S} = r_{orbit}$.

The satellite is the radius of the orbit. In this situation, $r_{M-S} = r_{orbit}$.

$$G\frac{m_M}{r_{M-S}} = v^2$$

$$m_M = \frac{v^2 r_{orbit}}{G} = \frac{(3.40 \times 10^3 \text{ m/s})^2 (3.83 \times 10^6 \text{ m})}{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2}$$

$$m_M = 6.64 \times 10^{23} \text{ kg}$$

(c)
$$U_{Total ME} = K + U_G$$

 $U_{Total ME} = \frac{1}{2} m_S v^2 - G \frac{m_M m_S}{r_{orbit}}$

$$U_{Total ME} = \frac{1}{2} (930 \text{ kg}) (3.4 \times 10^3 \text{ m/s})^2 - (6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2) \frac{(6.64 \times 10^{23} \text{ kg})(930 \text{ kg})}{3.83 \times 10^6 \text{ m}}$$

$$U_{Total ME} = -5.38 \times 10^9 \text{ J}$$

From (a) above, $m_M = \frac{v^2 r_{orbit}}{G}$, so if r_{orbit} decreases, v must decrease since m_M is a constant.

(e) According to the Law of Conservation of Angular Momentum,

$$I_1\omega_1 = I_2\omega_2$$

$$m_S r_1^2 \frac{v_1}{r_1} = m_S r_2^2 \frac{v_2}{r_2}$$
Note: $r_1 = r_M + Altitude$ and $r_2 = r_M + Altitude$.

$$v_1 = v_1 - r_2 v_2$$

 $v_2 = v_1 \frac{r_1}{r_2} = (3.40 \times 10^3 \text{ m/s}) \left(\frac{3.43 \times 10^6 \text{ m} + 3.71 \times 10^5 \text{ m}}{3.43 \times 10^6 \text{ m} + 4.36 \times 10^5 \text{ m}} \right)$

$$v_2 = 3.34 \times 10^3 \text{ m/s}$$

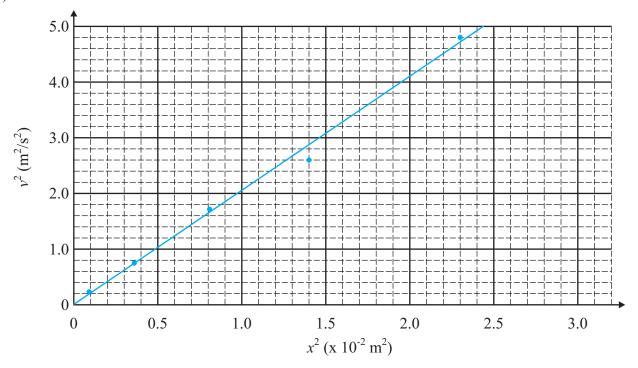
Mechanics 2007 C

3. (a)
$$EPE_1 = KE_2$$

 $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$$\frac{1}{2} (40 \text{ N} / \text{m}) x^2 = \frac{1}{2} m v^2$$

(b) i.



ii.
$$slope = \frac{\Delta y}{\Delta x} = \frac{\Delta v^2}{\Delta x^2} = \frac{\left(5.0 \text{ m}^2/\text{s}^2 - 0 \text{ m}^2/\text{s}^2\right)}{\left(2.4 \text{ x } 10^{-2} \text{ m}^2 - 0 \text{ m}^2\right)} = 2.0 \text{ x } 10^2 \text{ s}^{-2}$$
 Note: Since $\frac{1}{2} \left(40 \text{ N/m}\right) x^2 = \frac{1}{2} m v^2$, the slope of v^2 vs. v^2 should give
$$m = \frac{40 \text{ N/m}}{2.0 \text{ x } 10^2 \text{ s}^{-2}} = 0.2 \text{ kg}$$

$$\frac{40 \text{ N/m}}{m}, \text{ so } m = \frac{40 \text{ N/m}}{slope}.$$

$$m = 8.24 \text{ kg}$$

(d) i.
$$\frac{1}{2}$$
 (40 N/m) $x^2 + mg(h + x \sin\theta) = \frac{1}{2}mv^2$

ii.

No because now v^2 is not varying directly with x^2 , but is now varying with $x^2 + x$ as shown

below, which is not a linear relationship.

$$\frac{1}{2} \left(40 \text{ N/m} \right) x^2 + mg \left(h + x \sin \theta \right) = \frac{1}{2} m v^2$$

$$v^2 = \left(\frac{40 \text{ N/m}}{m} \right) x^2 + 2g \left(h + x \sin \theta \right)$$

$$v^2 = \left(\frac{40 \text{ N/m}}{m} \right) x^2 + 2gh + 2gx \sin \theta$$

$$v^2 = \left(\frac{40 \text{ N/m}}{m} \right) x^2 + \left(2g \sin \theta \right) x + 2gh$$

1. (a) Initially, there is no charge in the capacitor, therefore, no voltage across the capacitor. Thus, utilizing Kirchoff's loop rule indicating that the sum of a voltages in a complete loop within the circuit must be zero gives the following result.

$$\varepsilon - IR - V_C = 0$$
, where $V_C = \frac{Q}{C} = 0$ initially.
 $\varepsilon = IR = (2.25 \times 10^{-3} \text{ A})(550 \Omega)$

$$\varepsilon = 1.2 \text{ V}$$

(b)
$$\varepsilon - IR - V_C = 0$$

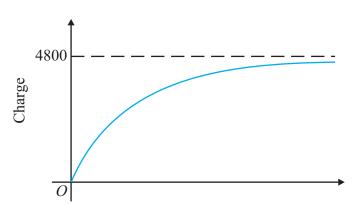
 $V_C = 1.2 \text{ V} - (4.0 \text{ x} 10^{-4} \text{ A})(550 \Omega)$
Or $V_C = \varepsilon \left(1 - e^{-\frac{t}{\tau}}\right)$, where $\tau = (550 \Omega)(4000 \text{ x} 10^{-6} \text{ F}) = 2.2 \text{ s}$.
Thus, $V_C = 1.0 \left(1 - e^{-\frac{t}{2.2}}\right)$. At $t = 4.0 \text{ s}$, $V_C = (12)\left(1 - e^{-\frac{4.0}{2.2}}\right) = 1.0 \text{ V}$.

(c)
$$V_C = \frac{Q}{C}$$
 Or $Q_C = C\varepsilon \left(1 - e^{-\frac{t}{\tau}}\right)$, where $C\varepsilon = \left(4000 \times 10^{-6} \text{ F}\right) \left(1.0 \text{ V}\right) = 4800 \text{ }\mu\text{C}$.

$$1.0 \text{ V} = \frac{Q_C}{4000 \times 10^{-6} \text{ F}}$$
 Thus, $Q_C = \left(4800\right) \left(1 - e^{-\frac{t}{2.2}}\right)$. At $t = 4.0 \text{ s}$, $Q_C = \left(4800\right) \left(1 - e^{-\frac{4.0}{2.2}}\right) = 4000 \text{ }\mu\text{C}$.

$$Q_C = 4000 \,\mu\text{C}$$

(d)



Graph of $Q_C = (4800)(1 - e^{-\frac{t}{2.2}})$

Time (s)

(e)
$$P = I^2 R = (4.0 \times 10^{-4} \text{ A})^2 (550 \Omega)$$

$$P = 8.8 \times 10^{-5} \text{ W}$$

(f) X Greater than Less than The same

The capacitance is increased by a factor of 3 which will decrease the time constant $\left(\tau = \frac{1}{RC}\right)$ by a

factor of 3. Thus, the rate of decrease of current will be decreased by a factor of 3 indicating that current will not decrease as rapidly. Therefore, the current will be greater at any given time (including t = 4.0 s) with the new capacitor than the original.

E & M 2007 C

2. (a) i.
$$\vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_o} = \frac{\rho_1 V}{\varepsilon_o}$$

$$E\left(4\pi r^2\right) = \frac{\left(\frac{+Q}{\left(\frac{4}{3}\pi a^3\right)}\right)\left(\frac{4}{3}\pi r^3\right)}{\varepsilon_o}$$

$$E = \frac{1}{4\pi\varepsilon} \frac{Qr}{a^3} = k\frac{Qr}{a^3}$$

ii.
$$\vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_o} = \frac{\rho_1 V}{\varepsilon_o}$$

$$E(4\pi r^2) = \frac{+Q}{\varepsilon_o}$$

$$E = \frac{1}{4\pi\varepsilon_o} + \frac{Q}{r^2} = k \frac{Q}{r^2}$$

iii.
$$\vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_o} = \frac{+Q + \rho_2 V}{\varepsilon_o}$$

$$E(4\pi r^2) = \frac{Q + \left(\frac{-Q}{\frac{4}{3}\pi(3a)^3 - \frac{4}{3}\pi(2a)^3}\right)^{\left[\frac{4}{3}\pi r^3 - \frac{4}{3}\pi(2a)^3\right]}}{\varepsilon_o}$$

$$E(4\pi r^2) = \frac{Q + \left\{\frac{-Q\left[\frac{4}{3}\pi(r^3 - 8a^3)\right]}{\frac{4}{3}\pi(27a^3 - 8a^3)}\right\}}{\varepsilon_o} = \frac{Q - \frac{Q(r^3 - 8a^3)}{(19a^3)}}{\varepsilon_o} = \frac{Q - \frac{Qr^3}{19a^3} + \frac{8Q}{19}}{\varepsilon_o} = \frac{Q + \frac{8Q}{19} - \frac{Qr^3}{19a^3}}{\varepsilon_o}$$

$$E(4\pi r^2) = \frac{\frac{19Q}{19} + \frac{8Q}{19} - \frac{Qr^3}{19a^3}}{\varepsilon_o} = \frac{\frac{27Q}{19a^3} - \frac{Qr^3}{19a^3}}{\varepsilon_o} = \frac{Q\left(27 - \frac{r^3}{a^3}\right)}{19\varepsilon_o}$$

$$E = \frac{1}{4\pi\varepsilon} \frac{Q}{19r^2} \left(27 - \frac{r^3}{a^3}\right)$$

iv.
$$\vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_o} = \frac{+Q - Q}{\varepsilon_o}$$

E = 0

(b) $dV = -\vec{E} \cdot d\vec{l} = 0$ because the electric field outside the spherical shell is a constant zero.

(c) Assume V_o is the potential on the outer sphere due to the charge on the outer sphere.

$$V = -\int \vec{E} \cdot d\vec{l} = -\int \frac{1}{4\pi\epsilon_{o}} \frac{Q}{r^{2}} dr = +\frac{1}{4\pi\epsilon_{o}} \frac{Q}{r} + C$$

$$V_{X} = \frac{1}{4\pi\epsilon_{o}} \frac{Q}{a} + V_{o}, \text{ while } V_{Y} = \frac{1}{4\pi\epsilon_{o}} \frac{Q}{2a} + V_{o}$$

$$V_{X} - V_{Y} = \left(\frac{1}{4\pi\epsilon_{o}} \frac{Q}{a} + V_{o}\right) - \left(\frac{1}{4\pi\epsilon_{o}} \frac{Q}{2a} + V_{o}\right) = \left(\frac{1}{4\pi\epsilon_{o}}\right) \left(\frac{Q}{a} - \frac{Q}{2a}\right) = \left(\frac{Q}{4\pi\epsilon_{o}}\right) \left(\frac{2}{2a} - \frac{1}{2a}\right) = \left(\frac{Q}{4\pi\epsilon_{o}}\right) \left(\frac{1}{2a}\right)$$

$$V_{X} - V_{Y} = \left(\frac{Q}{8\pi\epsilon_{o}a}\right)$$

2007 C

3. (a)

Clockwise X Counterclockwise

Applying Lenz's law, since the magnetic flux (through the closed path created by the U-shaped nichrome wire and the conducting bar) into the page is increasing as the conducting rod moves to the right, the current induced by this change will create a magnetic field out of the page to oppose the increase into the page. A counterclockwise current produces a magnetic field out of the page.

(b)
$$V = \frac{d\phi_B}{dt} = \frac{BdA}{\Delta t} = \frac{Bldw}{\Delta t} = BLv$$

$$I = \frac{V}{R} = \frac{BLv}{\lambda(L+2w)}$$

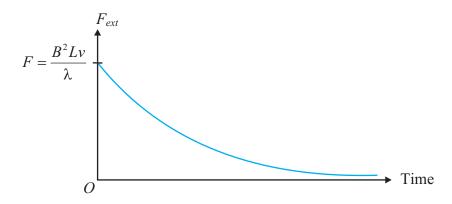
$$I = \frac{BLv}{\lambda(L+2vt)}$$

(c)
$$F = IlB = \left(\frac{BLv}{\lambda(L+2vt)}\right)LB$$

Note: Force of repulsion from parallel wire segment PQ is negligible.

$$F = \frac{B^2 L^2 v}{\lambda (L + 2vt)}$$

(d)



(e) ____Increases __X_Decreases ____Remains the same

Using the right hand rule, the force from the magnetic field on the current carrying wire is in the opposite direction of the motion of the conducting bar. Therefore, the bar will decelerate. The magnitude of this force is decreasing with time, but its direction is always to the left, opposite the direction the moving rod.

According to the right-hand rule, orient the fingers of the right hand in the direction of the current and rotate the wrist (keeping the fingers pointing in the direction of the current) until the fingers can curl in the direction of the magnetic field. The direction of the thumb indicates the direction of the magnetic force on the current carrying wire when these actions are performed.