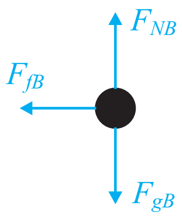
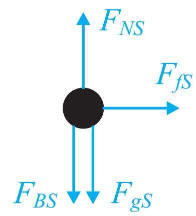
1. (a)

Block



Slab



(b) $M_B v_B + M_S v_S = (M_B + M_S) v_f$

$(0.50\text{ kg})\cdot(4.0\text{ m/s}) + 0 = (0.50\text{ kg} + 3.0\text{ kg}) v_f$

$\boxed{v_f = 0.57\text{ m/s}}$

Note: An alternative (but much longer) solution would be to use Newton's second law to get relative accelerations and calculate relative velocities. See last page for this solution.

(c) $W_{\text{NET}} = \Delta KE$

$F_f \cdot d = \frac{1}{2} M_s v_f^{\;2} - \frac{1}{2} M_s v^2$

$\mu F_{NB} \cdot d = \frac{1}{2} M_S \left(v_f^{\;2} - v^2 \right)$

$\mu M_B g \cdot d = \frac{1}{2} M_S \left(v_f^{\;2} - v^2 \right)$

$(0.20)(0.50\text{ kg})(9.8\text{ m/s}^2) d = \frac{1}{2}(3.0\text{ kg})\left[(0.57\text{ m/s})^2 - 0\right]$

$\sum F_y = F_{NB} - F_{gB} = M_B a$

$F_{NB} - M_B g = 0$

$F_{NB} = M_B g$

$\boxed{d = 0.500\text{ m}}$

Note: An alternative solution would be to use kinematic equations. See last page for this solution.

(d) $W_{\text{NET}} = \Delta KE$

$W_{\text{NET}} = \frac{1}{2} M_S \left(v_f^{\;2} - v^2 \right)$

$W_{\text{NET}} = \frac{1}{2}(3.0\text{ kg})\left[(0.57\text{ m/s})^2 - 0\right]$

$\boxed{W_{\text{NET}} = 0.49\text{ J}}$

Note: An alternative solution would be to use the definition of work. See last page for this solution.

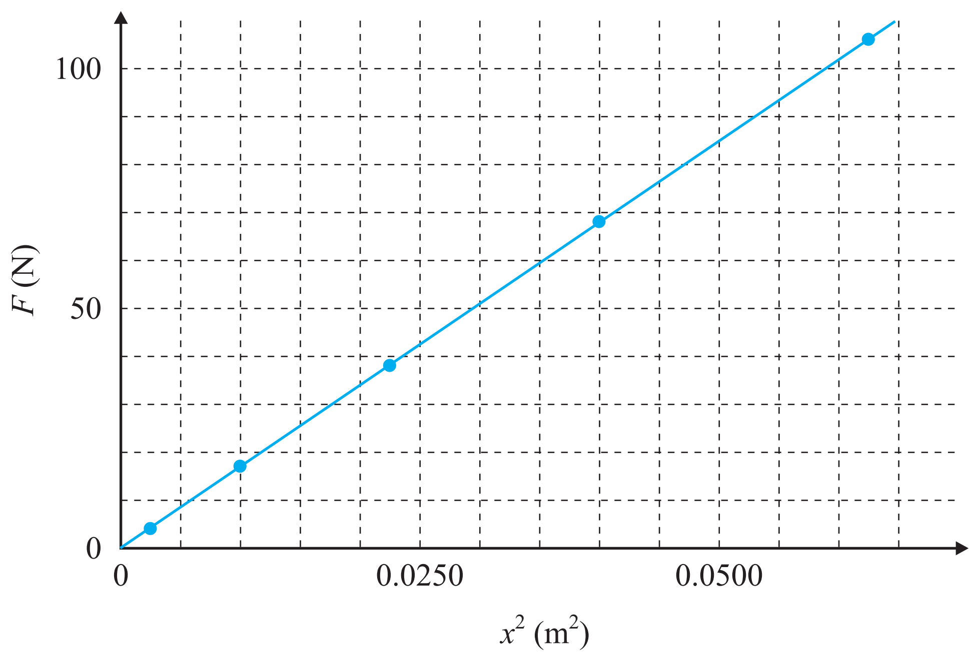
2. (a) Plot F vs. x^2

(b)

x (m)	F (N)	x^2 (m^2)
0.05	4	0.0025
0.10	17	0.0100
0.15	38	0.0225
0.20	68	0.04
0.25	106	0.0625

(c)

Force vs. Distance Compressed Squared



(d) $A = m = \frac{\Delta y}{\Delta x} = \frac{\Delta F}{\Delta x^2} = \frac{(106-4)\text{ N}}{(0.0625-0.0025)\text{ m}^2} = \boxed{1700\text{ N/m}^2 = A}$

(e) $W = F \cdot d\cos\theta = (Ax^2)\cdot(x)\cos 0 = \left[(1700\text{ N/m}^2)(0.10\text{ m})^2\right]\cdot(0.10\text{ m}) = \boxed{1.7\text{ J} = W}$

(f) $W = \Delta KE = \frac{1}{2}mv^2 - \frac{1}{2}mv_0{}^2$

$1.7\text{ J} = \frac{1}{2}(0.50\text{ kg})v^2 - \frac{1}{2}(0.50\text{ kg})(0\text{ m/s})^2$

$\boxed{v = 2.61\text{ m/s}}$

3. (a) $\Sigma F = F_{gx} - F_f = ma$

$mg\sin\theta - ma = ma$

$mg\sin\theta = 2ma$

$\boxed{a = \tfrac{1}{2} g\sin\theta}$

$\Sigma\tau = \tau_f = I\alpha$

$RF_f \sin\phi = \left(mR^2\right)\left(\frac{a_T}{R}\right)$ Note: $a_T = a$, $\phi = 90$, $\sin 90 = 1$

$F_f = ma$ (→ substituted into $\Sigma F = F_{gx} - F_f = ma$)

(b) $GPE_1 = TKE_2 + RKE_2$

$mgh = \tfrac{1}{2}mv_2^{\,2} + \tfrac{1}{2}I\omega_2^{\,2}$

$mgL\sin\theta = \tfrac{1}{2}mv_2^{\,2} + \tfrac{1}{2}\left(mR^2\right)\left(\frac{v_2}{R}\right)^2$

$gL\sin\theta = \tfrac{1}{2}v_2^{\,2} + \tfrac{1}{2}v_2^{\,2}$

$\boxed{v_2 = \sqrt{gL\sin\theta}}$

(c)

x	y
$x = ?$	$y_0 = \mathrm{H}$
$v_{x0} = v_2$	$y = 0$
	$v_{y0} = 0$
	$a_y = g$
	$t = ?$

$y = y_0 + v_0 t + \tfrac{1}{2}a_y t^2$

$H = 0 + 0 + \tfrac{1}{2}gt^2$

$t = \sqrt{2gH}$

$x = v_{x0}t = v_2 t$

$x = \left(\sqrt{gL\sin\theta}\right)\left(\sqrt{2gH}\right)$

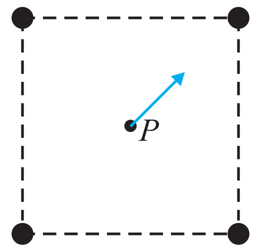
$\boxed{x = g\sqrt{2HL\sin\theta}}$

(d)

____ Less than ____ The same as __**X**__ Greater than

A disk has half the moment of inertia of the thin hoop resulting in a greater acceleration down the ramp and, thus, a greater velocity at the bottom of the ramp. Since the disk leaves the tabletop at a greater speed it will land a greater distance from the edge of the table than the thin hoop did.

1. (a)



(b) i. The electric field due to the charges in the upper left and lower right corner will cancel each other because they are of the same magnitude at point *P* and in opposite directions. The electric field due to the charges in the lower left (*LL*) and upper right (*UR*) corner are in the same direction (toward the upper right corner) at point *P*, therefore, adding to each other; and of the same magnitude.

$$E_{NET} = E_{LL} + E_{UR} = k\frac{Q}{r^2} + k\frac{Q}{r^2}$$

Note: $r = \frac{\sqrt{2}}{2}a$

$$E_{NET} = k\frac{Q}{\left(\frac{\sqrt{2}}{2}a\right)^2} + k\frac{Q}{\left(\frac{\sqrt{2}}{2}a\right)^2}$$

$$\boxed{E_{NET} = 4k\frac{Q}{a^2}}$$

ii. $V_{NET} = V_{LL} + V_{UL} + V_{UR} + V_{LR} = k\frac{Q}{r} + k\frac{-Q}{r} + k\frac{-Q}{r} + k\frac{-Q}{r} = -2k\frac{Q}{r} = -2k\frac{Q}{\frac{\sqrt{2}}{2}a}$

$$\boxed{V_{NET} = -\frac{4}{\sqrt{2}}k\frac{Q}{a}}$$

(c)

__X__ Positive ____ Negative ____ Zero

The positive charge at point *P* is being moved into a more positive region of space (closer to the lower left corner) and, therefore, against an increasingly stronger electric field. Thus, the movement of this charge will require an applied force (that will need to increase against the increasing electric field) to counter this electric field. At least a component of this force is in the direction of the displacement of the charge as it moves from point *P* to point *R*, so the work done $(W = FD\cos\theta)$ will be positive.

(d) i. Replace the charge at the lower left corner with charge of -*Q* as the electric field from this charge will cancel the electric field at the center from the charge in the upper right corner. The other two charges would also cancel each other for similar reasons as described above in 1. (b) i.

ii. Replace the charge at the upper left corner **OR** the lower right corner with a charge of +*Q* as this will create two positive charges and two negative charges all equidistance to the center of the square, so the net electric potential will be zero. However, the individual electric fields created by the four charges (each of which will be equal in magnitude) will not cancel because the direction of these electric fields are not in exactly opposite directions.

2. (a) Apply Kirchoff's loop rule

$$\varepsilon - IR_1 - \frac{Q}{C} = 0$$

$$\boxed{\varepsilon - \frac{dQ}{dt}R_1 = \frac{Q}{C}}$$

(b) $\frac{dQ}{dt}R_1 = \varepsilon - \frac{Q}{C}$

$\frac{dQ}{C\varepsilon - Q} = \frac{dt}{R_1 C}$ Now, Integrate both sides. Let $U = C\varepsilon - Q$, then $dU = -dQ$.

$$\int \frac{-dQ}{C\varepsilon - Q} = \int \frac{-dt}{R_1 C}$$

$\ln(C\varepsilon - Q) + L = -\frac{t}{R_1 C} + M$, where L and M are constants of integration. Let $N = M - L$.

$$\ln(C\varepsilon - Q) = -\frac{t}{R_1 C} + N$$

$C\varepsilon - Q = e^{-\frac{t}{R_1 C} + N}$, rearranging $C\varepsilon - Q = e^{-\frac{t}{R_1 C}} \cdot e^N$, or $C\varepsilon - Q = Be^{-\frac{t}{R_1 C}}$, where $B = e^N$ is a constant.

$Q = C\varepsilon - Be^{-\frac{t}{R_1 C}}$ To determine B, use the information that initially ($t = 0$), the charge, $Q = 0$.

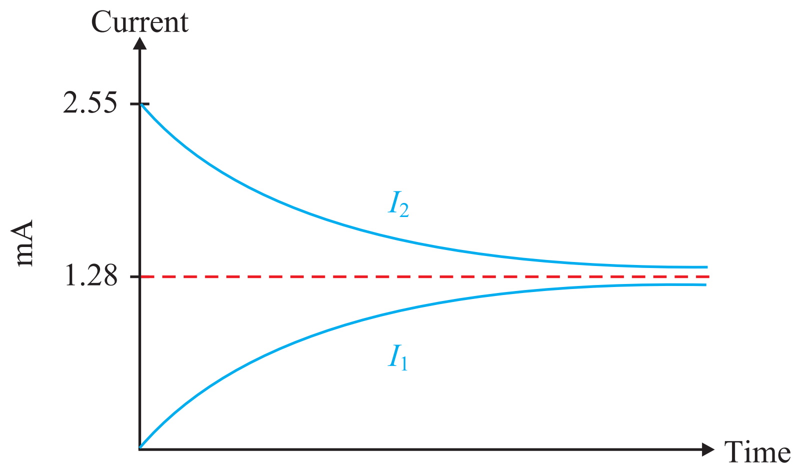
$0 = C\varepsilon - B$, or $B = C\varepsilon$ because $e^0 = 1$. Substituting this into the original expression for Q yields:

$Q = C\varepsilon\left(1 - e^{-\frac{t}{R_1 C}}\right)$, for the values given, this expression becomes $Q = (0.060\ \text{F})(12\ \text{V})\left[1 - e^{-\frac{t}{(4700\ \Omega)(0.060\ \text{F})}}\right]$

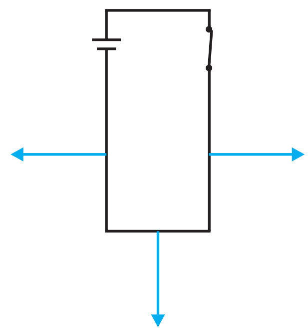
$$\boxed{Q = (0.72\ \text{C})\left[1 - e^{-\frac{t}{(282\ \text{s})}}\right]}$$

(c) $V = \frac{Q}{C}$ for the capacitor, so $4.0\ \text{V} = \frac{Q}{0.060\ \text{F}}$, or $Q = 0.24$ C. Substituting this value into the expression above yields: $0.24\ \text{C} = (0.72\ \text{C})\left[1 - e^{-\frac{t}{(282\ \text{s})}}\right]$. Now solve for t to get $\boxed{t = 310\ \text{s.}}$

(d)



3. (a)

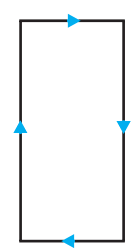


(b) $\sum F = F_S - F_B = ma = 0$

$kx = IwB_0$

$$\boxed{B_0 = \frac{kx}{Iw}}$$

(c) i.



ii. $\varepsilon = \frac{\Delta\phi}{\Delta t} = \frac{B_0 w \Delta l}{\Delta t} = B_0 w \frac{\Delta l}{\Delta t}$

$\varepsilon = B_0 w v_0$

$I = \frac{\varepsilon}{R}$

$$\boxed{I = \frac{B_0 w v_0}{R}}$$

(d) $P = I^2 R = \left(\frac{B_0 w v_0}{R}\right)^2 R$ **OR**

$P = \frac{W}{t} = \frac{Fd\cos\theta}{t}$ where $\sum F = F_A - F_B = ma = 0$

$P = F\cos 0 \frac{d}{t} = Fv$ where $F_A = F_B = IwB_0$

$P = IwB_0 v_0 = \left(\frac{B_0 w v_0}{R}\right) B_0 w v_0 = \frac{B_0^{\,2} w^2 v_0^{\,2}}{R}$

$$\boxed{P = \frac{B_0^{\,2} w^2 v_0^{\,2}}{R}}$$

(e)

__**X**__ Increases ____Decreases ____ Remains the same

As shown in the boxed derivation of part (d), the external force applied to the loop is directly promotional to the magnetic field strength.

1. (b) Alternate Solution:

Block

$$\sum F_x = F_{fB} = M_B a$$
$$-\mu F_{NB} = M_B a$$
$$-\mu M_B g = M_B a$$
$$a = -\mu g = \text{-}(0.20)\cdot\left(9.8\ \text{m/s}^2\right) = \text{-}1.96\ \text{m/s}^2$$

$$\sum F_y = F_{NB} - F_{gB} = M_B a$$
$$F_{NB} - M_B g = 0$$
$$F_{NB} = M_B g$$

$$v = v_0 + at$$
$$v_f = 4.0\ \text{m/s} + \left(\text{-}1.96\ \text{m/s}^2\right)t$$

Slab

$$\sum F_x = F_{fS} = M_S a$$
$$\mu F_{NB} = M_S a$$
$$\mu M_B g = M_S a$$
$$a = \mu \frac{\text{M}_\text{B}}{\text{M}_S} g$$
$$a = (0.20)\left(\frac{0.50\ \text{kg}}{3.0\ \text{kg}}\right)\left(9.8\ \text{m/s}^2\right) = 0.327\ \text{m/s}^2$$

$$v = v_0 + at$$
$$v_f = 0 + \left(0.327\ \text{m/s}^2\right)t$$

Set the two v_f expressions equal to each other and solve for time.

4.0 m/s - (1.96 m/s^2)t = (0.327 m/s^2)t

4.0 m/s = (2.287 m/s^2)t

t = 1.75 s

Now substitute this value of t in either expression of v_f to determine v_f.

$$v_f = 0 + \left(0.327\ \text{m/s}^2\right)t$$
$$v_f = \left(0.327\ \text{m/s}^2\right)(1.75\ \text{s})$$

v_f = 0.57 m/s

(c) $x = x_0 + v_0 t + \frac{1}{2}at^2$

x = 0 + 0 + ½(0.327 m/s^2)(1.75 m/s^2)

x = 0.50 m

(d) $W_{\text{NET}} = F_{\text{NET}}\cdot d = F_f \cdot d = \left(\mu M_B g\right)\cdot d$

$$W_{\text{NET}} = \left[(0.20)(0.50\ \text{kg})\left(9.8\ \text{m/s}^2\right)\right]\cdot(0.50\ \text{m})$$

W_{NET} = 0.49 J