1. (a) $GPE_1 = KE_2$

$m_1gL = \frac{1}{2}m_1v^2$

$$\boxed{v = \sqrt{2gL}}$$

(b) $\Sigma F = F_T - F_g = ma$

$F_T - m_1g = m_1(0\ m/s^2)$

$$\boxed{F_T = m_1g}$$

(c) $p_1 + p_2 = p_1' + p_2'$

$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$

$m_1v_1 + m_2v_2 = (m_1 + m_2)v'$ Inelastic Collision

$m_1\sqrt{2gL} + 0 = (m_1 + m_2)v'$

$$\boxed{v_2' = \left(\frac{m_1}{m_1 + m_2}\right)\sqrt{2gL}}$$

(d) $$\text{ratio} = \frac{KE_{sys}}{KE_{sys}'} = \frac{\frac{1}{2}m_1v_1^{\,2}}{\frac{1}{2}(m_1 + m_2)v'^2} = \frac{\frac{1}{2}m_1\left(\sqrt{2gL}\right)^2}{\frac{1}{2}(m_1 + m_2)\left\{\left[\frac{m_1}{(m_1 + m_2)}\right]\sqrt{2gL}\right\}^2}$$

$$\boxed{\text{ratio} = \frac{m_1 + m_2}{m_1}}$$

(e) From position A to position B the horizontal distance is L

The system experiences projectile motion from B to D in which the initial velocity is all horizontal.

x	y
$v_{xo} = \sqrt{2gL}$	$v_{yo} = 0\ m/s$
$x = ?$	$g = -9.8\ m/s^2$
	$y_o = L$
	$y = 0\ m$
	$t = ?$

$y = y_o + v_{yo}t + \frac{1}{2}at^2$

$0\ m = L + (0\ m/s)t + \frac{1}{2}gt^2$

$\frac{1}{2}gt^2 = L$

$$t = \sqrt{\frac{L}{\frac{1}{2}g}}$$

$$x = v_{xo}t = \left(\sqrt{2gL}\right)\cdot\left(\sqrt{\frac{L}{\frac{1}{2}g}}\right)$$

$x = 2L$

so the total horizontal distance from A to D is given by:

$x_{Total} = x_{A\text{-}B} + x_{B\text{-}D} = L + 2L$

$$\boxed{x_{Total} = 3L}$$

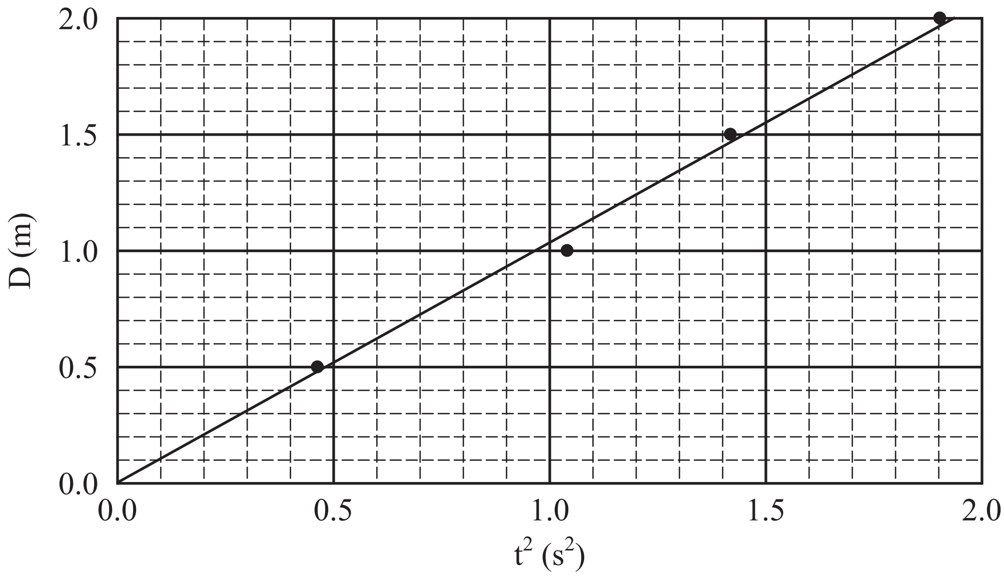
2. (a) $y = y_o + v_o t + \frac{1}{2}at^2$

$0 = D + 0 + \frac{1}{2}at^2$

$$\boxed{a = \frac{2D}{t^2}}$$

(b) i. D should be graphed as a function of t^2 to best determine the acceleration of the block because the relationship between D and t is $D \propto t^2$, so graphing D vs. t would give a parabola which would be difficult to analyze for acceleration. Graphing D vs. t^2 would yield a straight line (direct proportion). The slope of this graph would determine the proportionality constant for the relation between D and t^2 ($D \propto t^2$). From the kinematic equation $y = y_o + v_o t + \frac{1}{2}at^2$ which simplifies to $D = \frac{1}{2}at^2$ in this circumstance, the proportionality constant is one-half of the acceleration. Therefore, the slope of this graph (D vs. t^2) would equal one-half of the acceleration.

ii. 2



iii. slope, $m = \frac{\Delta y}{\Delta x} = \frac{\Delta D}{\Delta t^2} = \frac{(2.0\text{ m} - 0.00\text{ m})}{(1.91\text{ s}^2 - 0.00\text{ s}^2)} = 1.05\text{ m/s}^2$, but $m = \frac{1}{2}a$

$1.05\text{ m/s}^2 = \frac{1}{2}a$

$$\boxed{a = 2.10\text{ m/s}^2}$$

(c) $\Sigma F = F_T - F_g = ma$

$F_T - mg = ma$

$F_T = ma - mg = m(a - g)$

$\Sigma T = I\alpha$

$RF_T = I\frac{a}{R}$

$Rm(a - g) = I\frac{a}{R}$

$I = mR^2\left(\frac{a - g}{a}\right) = mR^2\left(1 - \frac{g}{a}\right)$

(d) The weight of the string was ignored which would reduce the experimental value of the rotational inertia. As the string unravels, the mass of the falling weight would increase, thus, increasing the measured acceleration, thereby, reducing the experimental rotational inertia.

3. (a) $dI = dmr^2 = \lambda dx r x^2$ $\qquad \lambda = \frac{m}{L} = \frac{dm}{dx}$

$$I = \int dI = \int_{-\frac{L}{3}}^{\frac{2L}{3}} \lambda x^2 dx = \lambda \frac{x^3}{3}\Bigg|_{-\frac{L}{3}}^{\frac{2L}{3}}$$

$$I = \lambda\left[\frac{\left(\frac{2L}{3}\right)^3}{3} - \frac{\left(\frac{-L}{3}\right)^3}{3}\right] = \frac{M}{L}\left[\frac{\left(\frac{8L^3}{27}\right)}{3} - \frac{\left(\frac{-L^3}{27}\right)}{3}\right] = \frac{M}{L}\left[\frac{8L^3}{81} + \frac{L^3}{81}\right] = \frac{M}{L}\left[\frac{9L^3}{81}\right]$$

$$\boxed{I = \tfrac{1}{9}ML^2}$$

(b) $GPE_1 = RKE_2$

$mgy_1 = ½I\omega_2$

$$Mg\frac{1}{6}L = \frac{1}{2}I\omega^2$$

$$Mg\frac{1}{6}L = \frac{1}{2}\left(\frac{1}{9}ML^2\right)\left(\frac{v}{r}\right)^2$$

$$gL = \frac{1}{3}\left(L^2\right)\left(\frac{v}{\frac{2}{3}L}\right)^2$$

$$gL = \frac{1}{3}\left(L^2\right)\frac{v^2}{\left(\frac{2}{3}L\right)^2}$$

$$g = \frac{1}{3}\left(L^2\right)\left(\frac{9}{4}\frac{v^2}{L^2}\right)$$

$$\boxed{v = \sqrt{\tfrac{3}{4}gL}}$$

Note: Use the lowest point of center of mass (cm) as 0 height, therefore, y_1, the initial height of the cm (located in the center of the rod), would be the distance the center of mass is from the pivot point. The cm is at $\frac{1}{2}L$ while the pivot point is at $\frac{2}{3}L$. The distance between these two points, y_1, is given by:

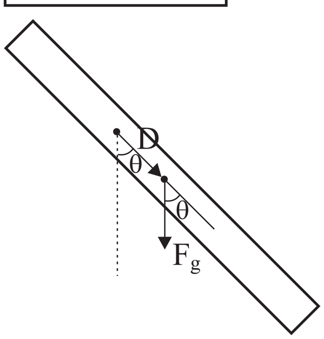
$$y_1 = \tfrac{1}{2}L - \tfrac{1}{3}L = \tfrac{3}{6}L - \tfrac{2}{6}L = \tfrac{1}{6}L$$

Note: r is the distance from the pivot point to the bottom of the rod when it is in the vertical position. This distance is $\frac{2}{3}L$.

(c) Period, T, for a physical pendulum is given by $T = 2\pi\sqrt{\frac{I}{MgD}}$ where D is the distance from the pivot point to the cm. For this rod, D is $\frac{1}{6}L$.

$$T = 2\pi\sqrt{\frac{I}{MgD}} = 2\pi\sqrt{\frac{\frac{1}{9}ML}{Mg\frac{1}{6}L}}$$

$$\boxed{T = 2\pi\sqrt{\frac{2L}{3g}}}$$



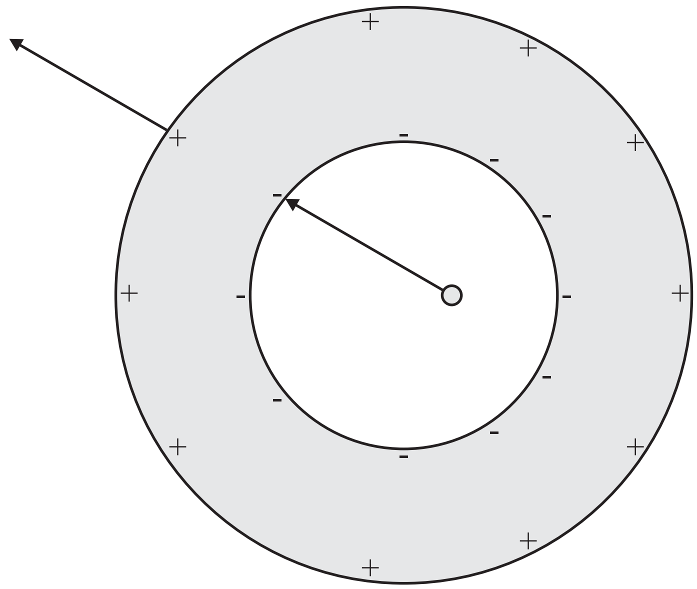
Note: A simple pendulum consists of a mass (bob) suspended from a string. A physical pendulum consists of a rigid body suspended body suspended by some point other than its cm such as the rod in this problem. When the body is displaced from its equilibrium position it will oscillate. When the angle of the displacement is small, the physical pendulum's oscillation approximates simple harmonic motion just as a simple pendulum does. The torque created by the physical pendulum is given by:

$T = -rF\sin\theta = I\alpha$ let r = D which represents the distance from the pivot point to the cm. For small angles, $\sin\theta \approx \theta$

$$-Dmg\theta = I\frac{d^2\theta}{dt^2}$$

$\frac{d^2\theta}{dt^2} = -\frac{MgD}{I}\theta$ so $\omega^2 = \frac{MgD}{I}$ and the period is given by $T = \frac{2\pi}{\omega}$.

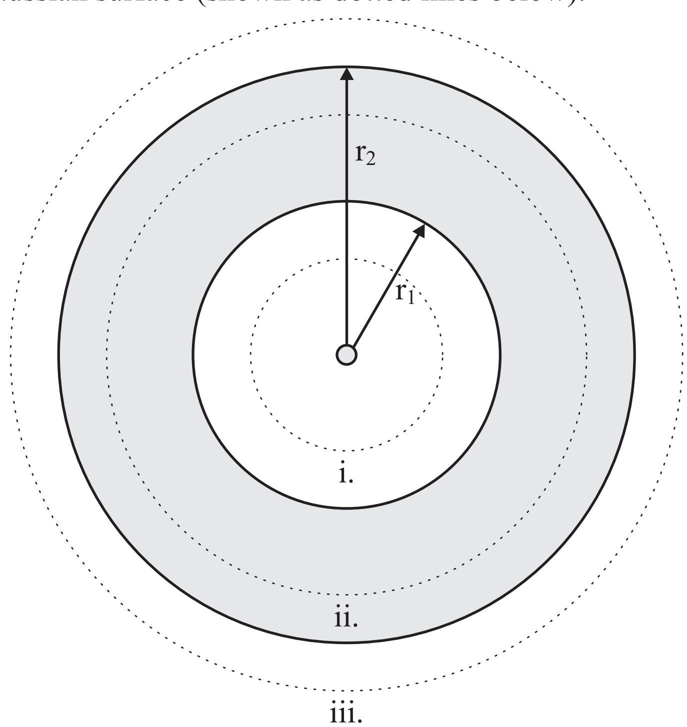
1. (a) i.
   ii.



(b)

1 V_a **3** V_b **5** V_c **1** V_d **3** V_e

(c) For a Gaussian surface, use a cylindrical shell with its center at the center of the infinite line charge for each of the following (i., ii., iii.). The electric field through the end faces of the cylindrical shell is 0. The only differences among i., ii., and iii. is the size of the Gaussian surface (shown as dotted lines below).



i. $\oint \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_o} = \frac{\lambda dl}{\varepsilon_o}$

$E2\pi r l = \frac{\lambda l}{\varepsilon_o}$

$$\boxed{E = \frac{1}{2\pi\varepsilon_o}\frac{\lambda}{r}}$$

ii. $\oint \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_o} = \frac{\lambda l + \rho V}{\varepsilon_o} = \frac{\lambda l + \rho A l}{\varepsilon_o}$

$E2\pi r l = \frac{\lambda l + \rho\left(\pi r^2 - \pi r_1^2\right)l}{\varepsilon_o}$

$$\boxed{E = \frac{1}{2\pi\varepsilon_o}\frac{\lambda + \rho\left(\pi r^2 - \pi r_1^2\right)}{r}}$$

iii. $\oint \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_o} = \frac{\lambda dl + \rho dV}{\varepsilon_o}$

$E2\pi r l = \frac{\lambda l + \rho\left(\pi r_2^2 - \pi r_1^2\right)l}{\varepsilon_o}$

$$\boxed{\text{Note: } \rho = \frac{Q}{V} = \frac{Q_2}{\left(\pi r_2^2 - \pi r_1^2\right)l} = \frac{\lambda_C}{\left(\pi r_2^2 - \pi r_1^2\right)} \text{ because } \lambda_C = \frac{Q}{l}}$$

$$E = \frac{1}{2\pi\varepsilon_o}\frac{\lambda + \frac{\lambda_C}{\left(\pi r_2^2 - \pi r_1^2\right)}\left(\pi r_2^2 - \pi r_1^2\right)}{r}$$

$$\boxed{E = \frac{1}{2\pi\varepsilon_o}\frac{\lambda + \lambda_C}{r}}$$

2. (a) **20 V** Voltage across the capacitor, C, is zero initially. The branch with the capacitor acts as a short circuit since charges will flow directly onto the top plate unimpeded (there is no charge, initially, on this plate so no repulsion to charges flowing onto this plate) and away from the bottom plate. Since the sum ov the voltages in any loop must be zero (Kirchoff's loop rule), there must be a voltage drop of 20 V (equivalent to EMF of battery) across R_2.

(b) **8 V** A long time after the switch is closed, the voltage across the C is 12 V which is the voltage across R_1 (parallel). Since the voltage drops in the circuit must add to the EMF of the battery (Kirchoff's loop rule), the voltage across R_2 would be 8 V (20 V - 12 V).

(c) A long time after the switch is closed, when the capacitor is fully charged, the branch with the capacitor acts as an open circuit, so no current flows into that branch. At this time, the current through R_2 is the same as that through R_1. Since the voltage across R_1 is 12 V, the current can be determined from Ohm's law: $I = \frac{V}{R} = \frac{12\text{ V}}{15{,}000\ \Omega} = 8.0 \times 10^{-4}\text{ A} = 0.80\text{ mA}$. Since this is the same current through R_2 which has a voltage of 8 V across it, the value of the resistance R_2 can be determined from Ohm's law.

$V_2 = IR_2$

$8\text{ V} = (8 \times 10^{-4}\text{ A})R_2$

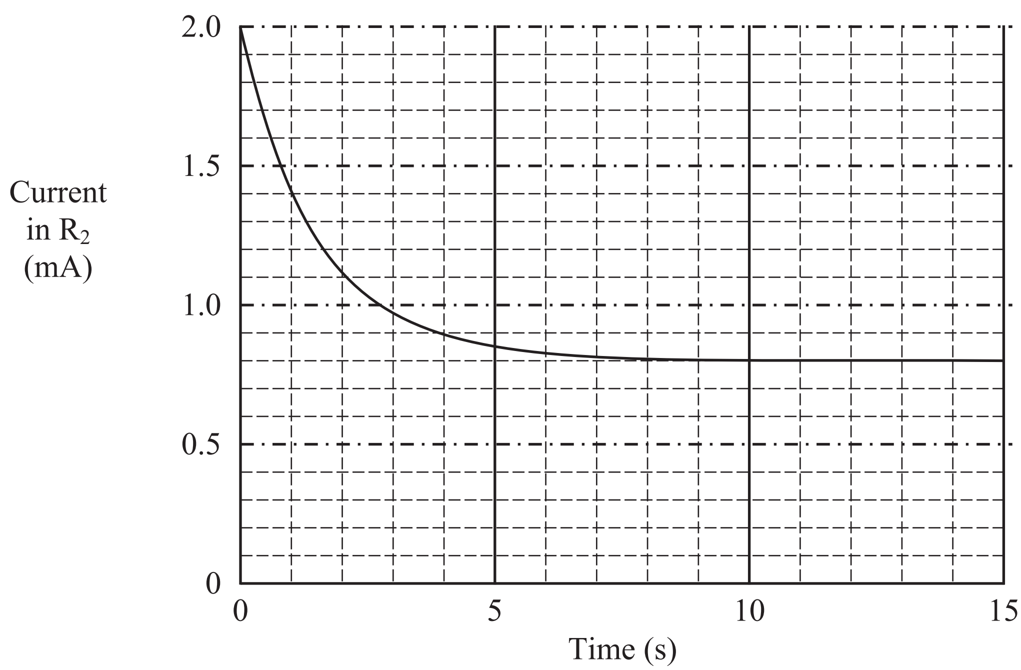
$$\boxed{R_2 = 10{,}000\ \Omega = 10\text{ k}\Omega}$$

(d) $E = ½QV$, but $Q = CV$, so $E = ½CV^2$

$E = ½(20\ \mu\text{F})(12\text{ V})^2$

$$\boxed{E = 1.44 \times 10^{-3}\text{ J} = 1.44\text{ mJ}}$$

(e)



Initially, with no charge on the capacitor, the branch with the capacitor acts as a short circuit with no resistance. So current through R_2 can be determined by using Ohm's law.

$I_o = \frac{V}{R} = \frac{20\text{ V}}{10{,}000\ \Omega}$

$I_o = 2.0 \times 10^{-3}\text{ A}$

$I_o = 2.0\text{ mA}$

It then would follow an exponential decay with a time constant of 1.5 s (the same as the voltage graph) to 0.80 mA (when the capacitor branch acts as an open circuit).

(f)

_____ Greater than _____ Less than __x__ The same as

The energy stored depends on the physical nature of the capacitor and the voltage across the capacitor which is the same as the battery after a long time. Decreasing the resistance of R_2 will only reduce the time needed to fully charge this capacitor and, thereby, reach full energy storage.

3. (a) $db = \frac{\mu_o}{2\pi}\frac{I}{r}dr$

$$B = \int_{4l}^{l} \frac{\mu_o}{2\pi}\frac{I}{r}dr = \frac{\mu_o}{2\pi}\frac{I}{r}\Bigg]_{4l}^{l} = \frac{\mu_o}{2\pi}I\left(\frac{1}{l} - \frac{1}{4l}\right) = \frac{\mu_o}{2\pi}I\left(\frac{4}{4l} - \frac{1}{4l}\right) = \frac{\mu_o}{2\pi}I\left(\frac{3}{4l}\right)$$

$$\phi_B = BA\cos\theta = \left[\frac{\mu_o}{2\pi}I\left(\frac{3}{4l}\right)\right]\left(12l^2\right)\cos(0)$$

$$\boxed{\phi_B = \frac{\mu_o}{2\pi}I(9l)}$$

(b)

_____ Clockwise   __x__ Counterclockwise

The current is decreasing exponentially with time causing the flux within the rectangular loop which is out of the page to decrease with time. Lenz's law states that the induced current will be in a direction such that the magnetic field created by this induced current will oppose the change. Since the current in the long straight wire is decreasing (exponentially) resulting in a decrease in flux out of the page, the induced current will travel counterclockwise producing a magnetic field out of the page to oppose the decrease caused by the decreasing current in the long straight wire.

(c) $$\frac{dI(t)}{dt} = \frac{d\left(I_o e^{-kt}\right)}{dt} = I_o e^{-kt}(-k) = -kI_o e^{-kt}$$

$$\varepsilon = \frac{d\phi_B}{dt} = \frac{\mu_o}{2\pi}\left(-kI_o e^{-kt}\right)(9l) = (-9kl)\frac{\mu_o}{2\pi}I_o e^{-kt}$$

$$I = \frac{\varepsilon}{R} = \frac{(-9kl)\frac{\mu_o}{2\pi}I_o e^{-kt}}{R}$$

$$\boxed{I = \left(\frac{-9kl}{R}\right)\frac{\mu_o}{2\pi}I_o e^{-kt}}$$

(d) $$P = I^2R = \left[\left(\frac{-9kl}{R}\right)\frac{\mu_o}{2\pi}I_o e^{-kt}\right]^2 R = \left(\frac{81k^2l^2}{R}\right)\left(\frac{\mu_o}{2\pi}\right)^2 I_o^2 e^{-2kt} = \left(\frac{81\mu_o^2k^2l^2}{4\pi^2R}\right)I_o^2 e^{-2kt}$$

$$dP = \frac{dE}{dt} \text{ so } dE = dP \cdot dt = \left(\frac{81\mu_o^2k^2l^2}{4\pi^2R}\right)I_o^2 e^{-2kt}dt$$

$$E = \int_0^\infty \left(\frac{81\mu_o^2k^2l^2}{4\pi^2R}\right)I_o^2 e^{-2kt}dt = \left(\frac{81\mu_o^2kl^2}{8\pi^2R}\right)I_o^2 e^{-2kt}\Bigg]_0^\infty = \left(\frac{81\mu_o^2kl^2}{8\pi^2R}\right)I_o^2\left[e^{-2k(\infty)} - e^{-2k(0)}\right]$$

$$E = \left(\frac{81\mu_o^2kl^2}{8\pi^2R}\right)I_o^2(0 - 1)$$

$$\boxed{E = \left(\frac{-81\mu_o^2kl^2}{8\pi^2R}\right)I_o^2}$$